+ 20 = 0

11. A line perpendicular to the line segment joining the points (1, 0) and (2, 3) divides it in the ratio 1: n. Find the equation of the line.

Solution:

We know that the coordinates of a point dividing the line segment joining the points (x_1, y_1) and (x_2, y_2) internally in the ratio m: n are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

$$\left(\frac{1(2)+n(1)}{1+n}, \frac{1(3)+n(0)}{1+n} \right) = \left(\frac{2+n}{1+n}, \frac{3}{1+n} \right)$$

We know that slope, $m = (y_2 - y_1)/(x_2 - x_1)$

$= (3 - 0)/(2 - 1)$

$= 3/1$

$= 3$

We know that two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.

Then, $m = (-1/m) = -1/3$

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0), if and only if, its coordinates satisfy the equation $y - y_0 = m (x - x_0)$

Here, the point is

$$\left(\frac{2+n}{1+n}, \frac{3}{1+n} \right)$$

$$\left(y - \frac{3}{1+n} \right) = \frac{-1}{3} \left(x - \frac{2+n}{1+n} \right)$$

$3((1 + n) y - 3) = (-(1 + n) x + 2 + n)$

$3(1 + n) y - 9 = - (1 + n) x + 2 + n$

$(1 + n) x + 3(1 + n) y - n - 9 - 2 = 0$

$(1 + n) x + 3(1 + n) y - n - 11 = 0$

∴ The equation of the line is $(1 + n) x + 3(1 + n) y - n - 11 = 0$.

12. Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point (2, 3).

Solution:

Given: the line cuts off equal intercepts on the coordinate axes i.e. $a = b$.

We know that equation of the line intercepts a and b on x-and y-axis, respectively, which is

$x/a + y/b = 1$

So, $x/a + y/a = 1$

$x + y = a$ … (1)

Given: point (2, 3)

$2 + 3 = a$

$a = 5$

Substitute value of 'a' in (1), we get

$x + y = 5$

$x + y - 5 = 0$

∴ The equation of the line is $x + y - 5 = 0$.

13. Find equation of the line passing through the point (2, 2) and cutting off intercepts on the axes whose sum is 9.

Solution:

We know that equation of the line making intercepts a and b on x-and y-axis, respectively, is $x/a + y/b = 1$. … (1)

Given: sum of intercepts = 9

$a + b = 9$

$b = 9 - a$

Now, substitute value of b in the above equation, we get

$x/a + y/(9 - a) = 1$

Given: the line passes through the point (2, 2),

So, $2/a + 2/(9 - a) = 1$

$[2(9 - a) + 2a] / a(9 - a) = 1$

$[18 - 2a + 2a] / a(9 - a) = 1$

$18/a(9 - a) = 1$

$18 = a (9 - a)$

$18 = 9a - a^2$

$a^2 - 9a + 18 = 0$

Upon factorizing, we get

$a^2 - 3a - 6a + 18 = 0$

$a (a - 3) - 6 (a - 3) = 0$

$(a - 3) (a - 6) = 0$

$a = 3$ or $a = 6$

Let us substitute in (1),

Case 1 (a = 3):

Then $b = 9 - 3 = 6$

$x/3 + y/6 = 1$

$2x + y = 6$

$2x + y - 6 = 0$

Case 2 (a = 6):

Then $b = 9 - 6 = 3$

$x/6 + y/3 = 1$

$x + 2y = 6$

$x + 2y - 6 = 0$

∴ The equation of the line is $2x + y - 6 = 0$ or $x + 2y - 6 = 0$.